

Higher algebras and mesonic spectrum in two-dimensional QCD

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Abstract

We construct composite operators in two-dimensional bosonized QCD, which obey a W_∞ algebra, and discuss their relation to analogous objects recently obtained in the fermionic language. A complex algebraic structure is unravelled, supporting the idea that the model is integrable. For singlets we find a mass spectrum obeying the Regge behavior.

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Two-dimensional gauge theories have always offered a good laboratory of ideas in Quantum Field Theory¹. Quantum chromodynamics (QCD₂), in particular, is an extraordinarily difficult problem, whose solution in terms of string ideas² has not been fully accomplished. Moreover, in spite of the fact that Abelian gauge theories are soluble (see [1] and references therein), the non-Abelian case has evaded a full solution, although a number of authors achieved considerable progress¹⁻⁸. Further understanding has been obtained in [9], where it has been proved that fermion bilocal operators obey a W_∞ -type algebra, opening a possibility of obtaining the full mesonic spectrum of the theory. In particular, $1/N$ corrections turn out to be feasible in such a scheme.

In a recent paper¹⁰, we have rewritten QCD₂ in terms of bosonic fields by integrating out the fermions, obtaining an integrable theory. Indeed, we start out of the generating functional

$$\mathcal{Z}[\eta, \bar{\eta}, i_\mu] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu e^{i \int d^2x \left[-\frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + \bar{\eta} \psi + \bar{\psi} \eta + i_\mu A^\mu \right]} , \quad (1)$$

with $D_\mu = \partial_\mu - ieA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$, η and $\bar{\eta}$ are the fermionic sources and i_μ is the gauge source. If we make the change of variables

$$A_+ = \frac{i}{e} U^{-1} \partial_+ U \quad , \quad A_- = \frac{i}{e} V \partial_- V^{-1} \quad , \quad (2)$$

it is possible to compute the fermion determinant in terms of the WZW action⁴ as

$$\det i \not{D} = e^{i\Gamma[UV]} \det i \not{\partial} \quad , \quad (3)$$

with

$$\Gamma[g] = \frac{1}{8\pi} \int d^2x \partial^\mu g^{-1} \partial_\mu g + \frac{1}{4\pi} \epsilon^{\mu\nu} \int dr \int d^2x \hat{g}^{-1} \dot{\hat{g}} \hat{g}^{-1} \partial_\mu \hat{g} \hat{g}^{-1} \partial_\nu \hat{g} \quad , \quad (4)$$

which obeys the Polyakov-Wiegman identity⁴

$$\Gamma[UV] = \Gamma[U] + \Gamma[V] + \frac{1}{4\pi} \text{tr} \int d^2x U^{-1} \partial_+ U V \partial_- V^{-1} \quad . \quad (5)$$

The fully bosonized form of QCD₂ is obtained using the invariance of the Haar measure, writing the fermionic determinant as

$$\det i \not{D} = \int \mathcal{D}g e^{iS_F[A,g]} \quad , \quad (6)$$

where

$$\begin{aligned} S_F[A, g] &= \Gamma[UgV] - \Gamma[UV] \\ &= \Gamma[g] + \frac{1}{4\pi} \int d^2x \left[e^2 A_\mu A_\mu - e^2 Ag \bar{A}g^{-1} - ieAg \bar{\partial}g^{-1} - ie\bar{A}g^{-1} \partial g \right] \quad . \end{aligned} \quad (7)$$

After some algebraic manipulations and defining $\tilde{g} = UgV$ we arrive at the result

$$\begin{aligned} \mathcal{Z}[\bar{\eta}, \eta, i_\mu] &= \int \mathcal{D}\tilde{g} e^{i\Gamma[\tilde{g}]} \int \mathcal{D}E \mathcal{D}U \mathcal{D}V \mathcal{D}(\text{ghosts}) \times \\ &\times e^{-i(c_V+1)\Gamma[UV] - i \int d^2x \text{tr} [\frac{1}{2}E^2 - \frac{1}{2}EF_{+-}] + iS_{\text{ghosts}} + i \int d^2x i_\mu A_\mu - i \int d^2x d^2y \bar{\eta}(x)(i\mathcal{D})^{-1}(x,y)\eta(y)} \end{aligned} \quad (8)$$

In order to obtain the above result, notice that the E -integration is Gaussian, reproducing the gauge field strenght squared, and the Casimir c_V is a consequence of the change of variables $\mathcal{D}A_+ \mathcal{D}A_- = \mathcal{D}U \mathcal{D}V e^{ic_V \Gamma[UV]}$ (or else the Dirac operator determinant (3) in the adjoint representation).

We can substitute the gauge field in terms of eq. (2) and try to rewrite the theory using the gauge-invariant combination $\Sigma = UV$, such as in

$$\text{tr} EF_{+-} = \frac{i}{e} \text{tr} U E U^{-1} \partial_+ (\Sigma \partial_- \Sigma^{-1}) \quad . \quad (9)$$

At this point we change variables as

$$U E U^{-1} = 2ie(c_V + 1) \frac{1}{4\pi} \partial_+^{-1} (\beta^{-1} \partial_+ \beta) \quad , \quad (10)$$

and obtain the final generating functional

$$\begin{aligned} \mathcal{Z}[\bar{\eta}, \eta, i_\mu] &= \int \mathcal{D}\tilde{g} e^{i\Gamma[\tilde{g}]} \mathcal{D}(\text{ghosts}) e^{iS_{\text{ghosts}}} \int \mathcal{D}\tilde{\Sigma} e^{-i(c_V+1)\Gamma[\tilde{\Sigma}]} \times \\ &\times \int \mathcal{D}\beta e^{i\Gamma[\beta] + i\frac{\lambda^2}{2} \text{tr} \int d^2x [\partial_+^{-1} (\beta^{-1} \partial_+ \beta)]^2} e^{i \int d^2x i_\mu A_\mu - i \int d^2x d^2y \bar{\eta}(x)(i\mathcal{D})^{-1}(x,y)\eta(y)} \end{aligned} \quad (11)$$

where $\lambda = \frac{c_V+1}{2\pi}e$. Here we distinguish three apparently independent sectors, namely the (conformally invariant) $\tilde{\Sigma}$ and \tilde{g} sectors, and the off-critically perturbed β sector. As discussed in ref. [10] (see also [12]) such sectors interact via the constraints arising from the BRST structure of the theory. The β -sector turns out to be integrable. Indeed, the β -equation of motion can be written in terms of

$$J_+^\beta = \beta^{-1} \partial_+ \beta \quad , \quad (12a)$$

$$J_-^\beta = 4\pi\lambda^2 \partial_+^{-2} J_+ \quad , \quad (12b)$$

as

$$[D_+, D_-] = [\partial_+ - J_+^\beta, \partial_- - J_-^\beta] = 0 \quad , \quad (12c)$$

or else as a non-linear conservation law

$$\partial_+ I_-^\beta = \partial_+ \left\{ 4\pi\lambda^2 J_-^\beta - \partial_+ \partial_- J_-^\beta + [J_-^\beta, \partial_+ J_-^\beta] \right\} = 0 \quad . \quad (13)$$

A duality-type transformation can be made in (11) in order to rewrite it in terms of fields appropriately describing the strong coupling (low-energy) limit. One uses

$$e^{\frac{i}{2}\lambda^2 \int d^2x [\partial_+^{-1} (\beta^{-1} \partial_+ \beta)]^2} = \int \mathcal{D}C_- e^{i \int d^2x [\frac{1}{2}(\partial_+ C_-)^2 + \lambda \text{tr} C_- \beta^{-1} \partial_+ \beta]} \quad , \quad (14)$$

and changes variables according to $C_- = \frac{1}{4\pi\lambda} W \partial_- W^{-1}$, arriving at

$$\mathcal{Z} = \int \mathcal{D}\tilde{\beta} e^{i\Gamma[\tilde{\beta}]} \int \mathcal{D}W e^{-i(c_V+1)\Gamma[W] - \frac{i}{2(4\pi\lambda)^2} \text{tr} \int d^2x [\partial(W\tilde{\partial}W^{-1})]^2} , \quad (15)$$

where $\tilde{\beta} = \beta W$. The W -equation of motion corresponds to an integrability condition similar to the β -formulation, and can be written as a conservation law in the form

$$\partial_+ I_-^W = \partial_+ \left\{ \frac{1}{4\pi} (c_V + 1) J_-^W - \frac{1}{(4\pi\lambda)^2} \partial_+ \partial_- \bar{J}_-^W - \frac{1}{(4\pi\lambda)^2} [J_-^W, \partial_+ J_-^W] \right\} = 0 \quad , \quad (16)$$

where $J_-^W = W \partial_- W^{-1}$.

In terms of (dual-equivalent)¹¹ theories (11) and (15), it is difficult to obtain the current algebra, due to the complicated interaction terms, namely the non-local β -interaction on the one hand, and the higher derivative W -interaction on the other hand. Therefore we introduce auxiliary fields, rendering in both formulations acceptable results, in the sense that it is possible to perform the canonical procedure. In such a case we have the β/W effective actions

$$S[\beta] = \Gamma[\beta] + \frac{1}{2} \int d^2x \text{tr} (\partial_+ C_-)^2 + \lambda \int d^2x \text{tr} C_- \beta^{-1} \partial_+ \beta \quad , \quad (17a)$$

$$S[W] = -(c_V + 1)\Gamma[W] + \frac{1}{2} \text{tr} \int d^2x \left[-B^2 + \frac{1}{2\pi\lambda} \partial_+ B \partial_- W W^{-1} \right] \quad . \quad (17b)$$

We note here the minus sign in front of the WZW term in the W -formulation (17b), signalling the presence of negative metric states. However this is not the full story. As a matter of fact, the complete system is described by the partition function (11/15), which as discussed in [10], based on the Karabali-Schnitzer argumentation¹², presents several constraints. The first-class constraints¹² select the Hilbert space, defining the appropriate cohomology, representing a GKO construction¹³. The theory thus defined has positive metric. In the present case there are also second-class constraints¹⁰. These are more complicated, and one is obliged to introduce the Dirac¹⁴ formulation in order to find the commutative algebras.

We start with the Poisson algebra. Using the formulation of ref. [6] we find (see also [10])

$$\Pi_- = \partial_+ C_- \quad , \quad (18a)$$

$$\tilde{\Pi}^\beta = \frac{1}{4\pi} \partial_0 \beta^{-1} + \lambda C_- \beta^{-1} \quad , \quad (18b)$$

where \sim means transposition with respect to the group indices, and $\tilde{\Pi}^\beta$ is the local part of the β canonical momentum (i.e. neglecting the WZW term), and satisfies

$$\{\beta_{ij}(t, x), \tilde{\Pi}_{kl}^\beta(t, y)\} = \delta_{il} \delta_{kj} \delta(x - y) \quad , \quad (19a)$$

$$\left\{ \tilde{\Pi}_{ji}^\beta(t, x), \tilde{\Pi}_{lk}^\beta(t, y) \right\} = -\frac{1}{4\pi} \left(\partial_1 \beta_{jk}^{-1} \beta_{li}^{-1} - \partial_1 \beta_{li}^{-1} \beta_{jk}^{-1} \right) \delta(x - y) \quad . \quad (19b)$$

The C_- equation of motion leads to its definition

$$C_- = \frac{1}{4\pi\lambda} J_-^\beta = \lambda \partial_+^{-2} (\beta^{-1} \partial_+ \beta) \quad . \quad (20)$$

On the other hand, for the W -formulation (dual) we have

$$\Pi_{ij}^W = \frac{\partial S}{\partial \partial_0 W_{ij}} = -\frac{1}{4\pi} (c_V + 1) \partial_0 W_{ji}^{-1} - \frac{1}{4\pi} (c_V + 1) A_{ji} + \frac{1}{4\pi\lambda} (W^{-1} \partial_+ B)_{ji} \quad , \quad (21a)$$

$$= \hat{\Pi}_{ij}^W - \frac{1}{4\pi} (c_V + 1) A_{ji} \quad , \quad (21b)$$

where A_{ij} is the contribution from the topological term to the momentum. There is no local representation for A_{ij} , but only its derivatives are necessary, i.e.

$$F_{ij;kl} = \frac{\delta A_{ij}}{\delta W_{lk}} - \frac{\delta A_{kl}}{\delta W_{ji}} = \partial_1 W_{il}^{-1} W_{kj}^{-1} - W_{il}^{-1} \partial_1 W_{kj}^{-1} \quad . \quad (21c)$$

The current and its $+$, $-$ derivatives are, in phase space, given by

$$J_-^W = W \partial_- W^{-1} = -4\pi\lambda \tilde{\Pi}_B \quad , \quad (22a)$$

$$\partial_+ J_-^W = -4\pi\lambda \partial_+ \tilde{\Pi}_B = 4\pi\lambda B \quad . \quad (22b)$$

The second-class constraints are obtained by coupling a subset of fields to an external gauge field A_+^{ext} for the β -formulation or A_-^{ext} for the W -formulation. One thus obtains two self-commuting constraints, but their difference is second-class, leading to the constraint

$$\Omega_{ij}^\beta = (\beta \partial_- \beta^{-1})_{ij} + 4\pi\lambda (\beta C_- \beta^{-1})_{ij} - (\tilde{g} \partial_- \tilde{g}^{-1})_{ij} \sim 0 \quad , \quad (23a)$$

or

$$\Omega^W = (c_V + 1) \Sigma^{-1} \partial_+ \Sigma - (c_V + 1) W^{-1} \partial_+ W + \frac{1}{\lambda} W^{-1} \partial_+ B W \sim 0 \quad , \quad (23b)$$

which in phase space do not depend on the auxiliary field, i.e.

$$\Omega_{ij}^\beta = 4\pi (\beta \tilde{\Pi}^\beta)_{ij} + \partial_1 \beta \beta^{-1} - 4\pi (\tilde{g} \tilde{\Pi}^{\tilde{g}})_{ij} - \partial_1 \tilde{g} \tilde{g}^{-1} \sim 0 \quad , \quad (24a)$$

$$\Omega^W = -\tilde{\Pi}^W W + \frac{1}{4\pi} W^{-1} \partial_1 W + \tilde{\Pi}^\Sigma \Sigma - \frac{1}{4\pi} \Sigma^{-1} \partial_1 \Sigma \sim 0 \quad . \quad (24b)$$

Only the currents

$$j_-^\beta = 4\pi \beta \tilde{\Pi}^\beta + \partial_1 \beta \beta^{-1} \quad , \quad (25a)$$

$$j_+^W = 4\pi \tilde{\Pi}^W W - W^{-1} \partial_1 W \quad , \quad (25b)$$

enter in the constraint, and they commute with the auxiliary fields, resp. (C_-, Π_-) , (B, Π_B) , as well as with the currents J_-^B , J_-^W , j_+^β and j_+^W . Therefore, for the latter objects Poisson and Dirac structures are the same. In particular

$$\left[J_{-ij}^\beta(x), \partial_+ J_{-kl}^\beta(y) \right] = (4\pi\lambda)^2 i \delta_{kj} \delta_{il} \delta(x^1 - y^1) \quad , \quad (26a)$$

and

$$\left[J_{-ij}^W(x), \partial_+ J_{-kl}^W(y) \right] = (4\pi\lambda)^2 i \delta_{kj} \delta_{il} \delta(x^1 - y^1) \quad . \quad (26b)$$

Observe the duality of the phase space, where the role of (C_-, Π_-) is interchanged (in inverse order!) with that of $(\Pi_B, -B)$.

We define $J(x) = \frac{1}{4\pi\lambda} J_-(x)$, for both $(J_-^{\beta, W})$, the bilocal

$$M(x, y) = \text{tr } J(t, x) \partial_+ J(t, y) \quad , \quad (27)$$

and we are led to the W_∞ -algebra

$$[M(x, y), M(z, w)] = i\delta(x - w)M(z, y) - i\delta(z - y)M(x, w) \quad . \quad (28)$$

Time evolution may be obtained from the Hamiltonian, which in the β -formulation reads

$$H_\beta = -\frac{1}{16\pi} \left[\left(J_+^\beta \right)^2 + \left(j_-^\beta \right)^2 \right] - \frac{1}{2} \lambda J_+^\beta C_- + \pi \lambda^2 C_-^2 + \frac{1}{2} \Pi (\Pi_- - 2C') \quad . \quad (29)$$

We still have to take into account the constraint, which eliminates j_- in terms of the \tilde{g} fields. This procedure is safe, once j_- commutes with the variables considered before, namely C_- , Π_- , J_+ and J_- . The commutators of the Hamiltonian with $M(x, y)$ leads to

$$-i[H, M(x, y)] = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) M(x, y; t) - P(x, y; t) - \lambda Q^{(1)}(x, y; t) \quad , \quad (30)$$

where

$$P(x, y; t) = \text{tr } \partial_+ J(t, x) \partial_+ J(t, y) \quad , \quad (31a)$$

$$Q^{(1)}(x, y; t) = \text{tr } J(t, x) \partial_+^2 J(t, y) \quad . \quad (31b)$$

In terms of the phase-space variables, we have

$$P(x, y; t) = \text{tr } \Pi(t, x) \Pi(t, y) \quad , \quad (32a)$$

$$Q^{(1)}(x, y; t) = \text{tr } C_-(t, x) J_+(t, y) \quad , \quad (32b)$$

where

$$J_+(t, x) = j_+(t, x) + 4\pi\lambda C_-(t, x) \quad . \quad (33)$$

If we try to include such fields in the $M(x, y)$ algebra, we generate an infinite number of terms. We computed

$$-i[P(x, y), M(z, w)] = -P(x, w)\delta(y - z) - P(y, w)\delta(x - z) \quad , \quad (34)$$

which closes, but

$$-i[Q^{(1)}(x, y; t), Q^{(1)}(z, w; t)] = Q^{(2)}(x, z; y; t)\delta(y - w) - Q^{(2)}(z, x; y; t)\delta(y - w) \quad , \quad (35)$$

up to contact terms, where $Q^{(2)}(x, z; y; t) = \text{tr } C_-(x)C_-(z)J_+(y)$. Moreover we find also

$$\begin{aligned} -i[P(x, y), Q^{(1)}(z, w)] &= -4\pi\lambda [M(z, y)\delta(x - w) + M(z, x)\delta(y - w)] - \\ &\quad - R(y, w)\delta(x - z) - R(x, w)\delta(y - z) \quad , \end{aligned} \quad (36)$$

where $R(x, y) = \text{tr } \Pi(x)J_+(y)$. In fact, here, there are infinite new terms, which get generated by multiple commutators, such as $Q^{(n)}(\{z_i\}, w) = \text{tr } C_-(z_1) \cdots C_-(z_n)J_+(w)$, as well as analogous $R^{(n)}(\{z_i\}, w)$ terms.

Such W_∞ -type algebras appear not only in QCD₂⁹ but also in the description of the incompressible fluid of the Quantum Hall effect¹⁵, where however only singlets are considered.

If we sum over all symmetry group indices, i.e. consider only singlets, a further simplified structure arises. Taking the trace of (13) [or equivalently (16)], we have the conserved charges

$$Q^\beta = \text{tr} \int dx^1 \left\{ 4\pi\lambda^2 J_-^\beta - \partial_+ \partial_- J_-^\beta \right\} \quad , \quad (37a)$$

$$Q^W = \text{tr} \int dx^1 \left\{ 4\pi\lambda^2 (c_V + 1) J_-^W - \partial_+ \partial_- J_-^W \right\} \quad . \quad (37b)$$

We can use the phase-space formulation for

$$J_-^\beta = 4\pi\lambda C_- \quad , \quad (38a)$$

$$\partial_+ \partial_- J_-^\beta = 4\pi\lambda^2 j_+^\beta + (4\pi\lambda)^2 \lambda C_- \quad , \quad (38b)$$

obtaining

$$Q^\beta = -4\pi\lambda^2 \int dx^1 \text{tr } j_+^\beta \quad , \quad (39)$$

with $j_+^\beta = -4\pi\tilde{\Pi}^\beta \beta + \beta^{-1}\beta'$.

The algebraic structure just discussed is not the only higher algebra underlying the theory. As we discussed in a previous publication, the currents $I_-(x^-)$ obey an affine Lie structure given by¹⁶

$$\left\{ I_-^{ij}(x), I_-^{kl}(y) \right\} = (\delta^{il}\delta^{kj} - \delta^{kj}\delta^{il}) \delta(x - y) + \frac{c_V + 1}{2\pi} \delta_{il}\delta_{kj} \delta'(x - y) \quad , \quad (40)$$

and $\widehat{J}_-(x^+, x^-)$ is a realization of such algebra,

$$\left\{ I_-^{ij}(x), \widehat{J}_-^{kl}(y^+, y^-) \right\} = \left(\delta^{il} \widehat{J}_-^{kj} - \delta^{kj} \widehat{J}_-^{il} \right) \delta(x^- - y^-) + \delta_{il} \delta_{kj} \delta'(x^- - y^-) \quad , \quad (41)$$

where \widehat{J}_- is either J_-^W or J_-^β with a suitable normalization factor. Therefore, $\partial_+ \widehat{J}_-$ is a primary field of the I_- affine algebra^{16,17}, depending on x^+ as a parameter.

We should however stress the fact that while I_- is a *conserved* current ($\partial_+ I_- = 0$), the other higher operators are not. Therefore their action generates new states of the theory.

The presence of higher conservation laws, and the complex algebraic structure is characteristic of integrable systems^{18–20}, confirming recent claims that two-dimensional QCD^{20,10}, or else high-energy scattering of strongly interacting systems²¹ are described by integrable quantum field theories^{22,23}.

The problem simplifies drastically upon consideration of singlets only. Indeed, as discussed above, $\text{tr } j_+$ is a right-moving field. The $(++)$ component of the energy momentum tensor is given by

$$\begin{aligned} T_{++} &= \frac{1}{N+1} \left\{ -\frac{1}{16\pi} (J_+)^2 + \frac{1}{2} (\partial_+ C_-)^2 \right\} \\ &= \frac{1}{N+1} \left\{ -\frac{1}{16\pi} (j_+)^2 - \frac{1}{2} \lambda j_+ C_- - \pi \lambda^2 C_-^2 + \frac{1}{2} (\partial_+ C_-)^2 \right\} \quad , \end{aligned} \quad (42)$$

where we introduced the factor $\frac{1}{N+1}$ in order for the limit $N \rightarrow \infty$ to be well defined, in accordance with the Sugawara construction¹⁶.

Consideration of the trace part leads to a left-moving C_- field and the last term above drops off. Therefore we are left with the mode expansions

$$j_+^{\text{tr}} = i\lambda \sum j_n e^{in\lambda(x-t)} \quad , \quad (43a)$$

$$C_-^{\text{tr}} = i \sum C_n e^{in\lambda(x-t)} \quad , \quad (43b)$$

with the following commutation relation for the modes

$$[j_n, j_m] = 4n\delta_{n,-m} \quad , \quad (44a)$$

$$[C_n, C_m] = \frac{1}{2\pi n} \delta_{n,-m} \quad . \quad (44b)$$

The integral of (42) is given by

$$\begin{aligned} T_{++}^{(0)} &= \frac{\lambda^2}{4(N+1)} \sum_{n \geq 1} j_{-n} j_n + \frac{\lambda^2}{8(N+1)} j_0^2 + \frac{\pi}{(N+1)} \lambda^2 \sum (j_{-n} C_n + C_{-n} j_n) \\ &\quad + \frac{4\pi^2 \lambda^2}{N+1} \sum C_{-n} C_n + \frac{2\pi^2 \lambda^2}{N+1} C_0^2 \quad . \end{aligned} \quad (45)$$

The zero-mode component can be interpreted as mass squared up to an undetermined constant, and

$$\frac{\lambda^2}{N+1} = \frac{e^2(N+1)}{(2\pi)^2} = \frac{e_{fin}^2}{(2\pi)^2} \quad , \quad (46)$$

where e_{fin} is defined as a new coupling constant³ in the limit $N \rightarrow \infty$. We thus find

$$T^{(0)} j_{-n} |0\rangle = \frac{\lambda^2}{N+1} (n j_{-n} + 4n\pi C_{-n}) |0\rangle \quad , \quad (47a)$$

$$T^{(0)} C_{-n} |0\rangle = \frac{\lambda^2}{N+1} \left(\frac{1}{2n} j_{-n} + \frac{2\pi}{n} C_{-n} \right) |0\rangle \quad . \quad (47b)$$

The eigenvectors are of the form $j_{-n} + \xi C_{-n}$, and we obtain

$$\xi_{\pm} = -n \left(n - \frac{2\pi}{n} \right) \left[1 \pm \sqrt{1 + \frac{8\pi}{\left(n - \frac{2\pi}{n} \right)^2}} \right] \quad . \quad (48)$$

For large values of n , we have

$$\xi_+ = -2(n^2 - 2\pi) \quad , \quad (49a)$$

$$\xi_- = 4\pi \quad , \quad (49b)$$

for which we find, in the first case, mass eigenvalues of the type

$$m^2 \sim e_{fin}^2 \times \mathcal{O} \left(\frac{1}{n^5} \right) \quad , \quad (50a)$$

or

$$m^2 \sim n e_{fin}^2 \quad , \quad (50b)$$

therefore, the former is degenerate, while the latter presents a Regge behaviour, in accordance with ref. [3]. Thus it is possible in principle to find 't Hooft's results³, as well as corrections to it. Relations with collective fields are yet to be discovered²⁴.

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